Dynamic Site Index Equation for Thinned Stands of Even-Aged Natural Longleaf Pine

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ABSTRACT

A dynamic site equation derived using the generalized algebraic difference approach was developed for thinned stands of natural longleaf pine (*Pinus palustris* Mill.) in the East Gulf region of the United States using 40 years of measurements on 285 permanent plots. The base model predicts height growth of trees once they reach 4.5 ft and was fit using a varying parameter for each tree and global parameters that are constant for all 3,267 trees. Parameters were estimated in one step using the dummy variable approach and a first-order autoregressive error term to account for serial correlation. The final base-age invariant equation allows the user to specify the number of years required for trees to reach 4.5 ft in height.

Keywords: *Pinus palustris*, base-age invariant, grass stage, site quality, height growth, natural stands

Longleaf pine (*Pinus palustris* Mill.) is managed as both natural seeded and planted stands in the southeastern United States. Modeling of early height development is complicated by the grass stage period, during which no height growth occurs. The length of time between germination and seedling height initiation in natural stands usually lasts 4–5 years but may range from 1 to 20 years (Croker and Boyer 1975), and it is followed by a period of rapid height growth. Emergence from the grass stage can depend on seedling quality for planted stands, intraspecific competition for natural stands, interspecific competition for both planted and natural stands, insect and disease incidence, and climate factors (Croker and Boyer 1975, Lauer 1987, Barnett 1989, Haywood 2005). Silvicultural practices that reduce competition and improve soil conditions can reduce duration of the grass stage. The incidence of brown-spot needle blight caused by *Mycosphaerellaceae dearnessii* M.E. Barr varies by geographic location and can increase the number of years a tree remains in the grass stage period (Kais et al. 1986).

This project models height growth of thinned, natural even-aged longleaf stands regenerated by the shelterwood method using the Regional Longleaf Pine Growth Study (RLGS) (Farrar 1978). Stands are considered even-aged because they were established by methods completed with final overstory removal, but trees may vary in both age and year of height growth initiation. Previous site index curves developed for longleaf pine using the RLGS data have been in the form of anamorphic or simple polymorphic models. Farrar (1981) used data from the first and second remeasurements to develop an equation that used age in a fourth-order polynomial equation. Rayamajhi et al. (1999) updated this equation with data from the sixth remeasurement (30 years) and compared its performance with a Chapman-Richards type function (Carmean 1972). They found that the Chapman-Richards model had a higher mean square error compared with the updated Farrar model and performed poorly in younger and older age classes. Both efforts defined age as ring count at dbh plus 7 years to account for the grass stage period and initial height development.

Site index curves have also been developed for other specific populations of longleaf pine and have been used to compare height development for different site classifications. Most models have been anamorphic. These include graphical curves for second-growth natural longleaf pine (US Forest Service 1929) with age based on ring count at dbh plus 7 years, an equation for natural stands (Schumacher and Coile 1960) that assumes a 2-year grass stage, and a base-age invariant Chapman-Richards equation for young longleaf pine plantations in southwest Georgia (Brooks and Jack 2006) based on plantation age. A base-age invariant polymorphic equation was developed for direct seeded longleaf pine in Louisiana (Cao et al. 1997) with age based on year seeded. Boyer (1983) used a Schumacher type model (Schumacher 1939) to compare height growth patterns of young longleaf pine among old field, prepared forest, and unprepared forest sites. None of these models explicitly account for length of the grass stage.

Site index is used as an estimate of site productivity, is used to predict future height growth, and is an important input in most growth and yield models. Site index models have not been known for accurate characterization of early stand development when tree growth can be strongly influenced by interspecific competition and when "average" tree growth may be strongly influenced by short-term climatic events. Models are selected to have logical patterns of growth in early stand development but prediction of site index in young stands is usually found to have a large error component. Longleaf is problematic in this respect because site index models have not treated the grass stage as a discrete event but as continuous slow growth over a period that should not be used for projection. Years in the grass stage may be treated as an assumed constant to provide an estimate of stand age, or when establishment age is known, the model will indirectly account for the grass stage period as an average period for the sample population.

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Table 1. Distribution of plot measurements by average breast height age (age_{bh}), and average total height of dominant and codominant sample trees.

Age_{bh} (years)	Number of plots: Total height (ft) 6–15	16–35	36–55	56–75	76–95	96–120	Total
<21	17	112	236	52			417
21–40			49	364	120		533
41–60			17	116	285	17	435
61–80				75	246	55	376
81–100				38	127	37	202
101–120				11	29	11	51
Total	17	112	302	656	807	120	2014

Table 2. Distribution of plot measurements by average breast height age (age_{bh}), and year.

Year	Age_{bh} (years) <21	21–40	41–60	61–80	81–100	101–120	Total
1964–1968	57	45	55	22			179
1969–1973	92	39	62	33	2		228
1974–1978	57	48	65	37	6		213
1979–1983	15	54	54	39	20		182
1984–1988	60	60	40	47	27		234
1989–1993	51	77	32	59	32	2	253
1994–1998	59	60	40	64	34	6	263
1999–2003	19	70	50	47	41	18	245
2004–2007	7	80	37	28	40	25	217
Total	417	533	435	376	202	51	2014

The accurate description of early height growth in longleaf pine is becoming more important with restoration initiatives over a diverse landownership. Comparison of management options and management interventions may increase the value of this species to landowners. The difficulty is that longleaf pine may remain in the grass stage for years and then achieve rapid, near-linear height growth once taller than 4.5 ft for 10 years. An error of 1 year in time of emergence during this linear growth period could result in a 2–4-ft error in age–height estimates.

The objective of this project was to develop an updated site index equation using the RLGS data set that will better estimate height growth in stands less than 25 years (although use of young stand estimates will still have limitations) and provide an improved estimate of site productivity by accounting for the discrete grass stage. The concept is that a measure of site productivity based on height growth in stands past the grass stage will be more accurate than a measure of site productivity that includes length of time in the grass stage (which is dependent on inter- and intraspecific competition, microsite, seedling quality etc.). Model selection considered whether data support anamorphic (proportional) or polymorphic (not proportional with different shapes) models. The model functional form was important to ensure logical outcomes throughout an age range of over 100 years. It was also desirable to use a base-age invariant model so that the system of curves provide logical results when the base age is algebraically changed to be useful in a given management context. Statistical methods must account for the unique stand structure of natural stands, account for the pooled cross sectional and longitudinal structure of the data, and address the difficulty of determining stand age for longleaf pine.

Data

Data for this project were collected on permanent measurement plots since 1964 by Auburn University, Mississippi State University, and other public owners as part of a Forest Service Southern Station cooperative study investigating production of thinned, even-aged, naturally regenerated stands in the East Gulf region of the southern United States. Plots were initially selected to fill an array of cells with five 20-year age classes, five 10-ft site index classes, and five 30-ft^2 basal area classes (Farrar 1979, 1993). Plots were measured every 5 years, with some exceptions, and additional plots were added as the study progressed. A total of 2,014 plot measurements were completed covering a wide range of age and height classes (Table 1). Plot measurements were distributed reasonably well across age class and measurement years except that stands over 75 years old increased in number as the study progressed (Table 2).

A subsample of dominant and codominant trees were measured on 0.1- or 0.2-ac permanent plots. At the time of plot installation, every fifth tree in each 1-in. dbh class was selected to be measured for height and cored to determine age if the tree was dominant or codominant. When possible, there were at least 2 sample trees per dbh class and a minimum of 10 sample trees per plot unless there were fewer than 10 trees. Plots were periodically thinned and plots were added as the study progressed such that the number of measurements per tree differed by plot and could differ by tree within a plot. Dominant and codominant trees selected from this population of sampled trees had to be measured at least three times and at least half as many times as the number of all measurements taken on a given plot, and they must have been measured at the last or second-to-last plot measurement. Furthermore, the tree must have been classified as dominant or codominant for all measurement periods. This resulted in a data set with total of 19,527 measurements on 3,267 trees distributed over 285 plots. The number of measurements varied per tree, with approximately one-third of trees being measured three or four times, another third being measured five to seven times, and the remaining third being measured eight or more times.

Stand age is not simply defined for this natural stand data set. Previous site index functions for this data set used average ring count just below dbh plus 7 years on dominant and codominant trees as age for each plot (Farrar 1981). Seven years has traditionally been used to account for the number of years longleaf pine takes to emerge from the grass stage and reach 4.5 ft. Ring count varies for trees within a plot at any given measurement. Only 25% of plots had a ring count range of 3 years or less. The ring count interquartile range was 3–8 years. Less than 5% of plots had ring count differences greater than 13 years. Trees were considered residuals from the previous stand and excluded if they were greater than 10 years older than the average plot age. For this article, the variable breast height age (age_{bh}) is defined as ring count just below 4.5 ft.

Model Selection

Preliminary examination of height series and trial fits of many models determined that height growth was best modeled using a polymorphic system. Observed growth patterns were that initial longleaf height growth was nearly linear and rapid once trees are greater than 4.5 ft tall; that the relationship between height and site quality was polymorphic but became more proportional at ages greater than 40 years; and that longleaf continued to grow in height, albeit at a very slow rate, at ages greater than 90 years. This required

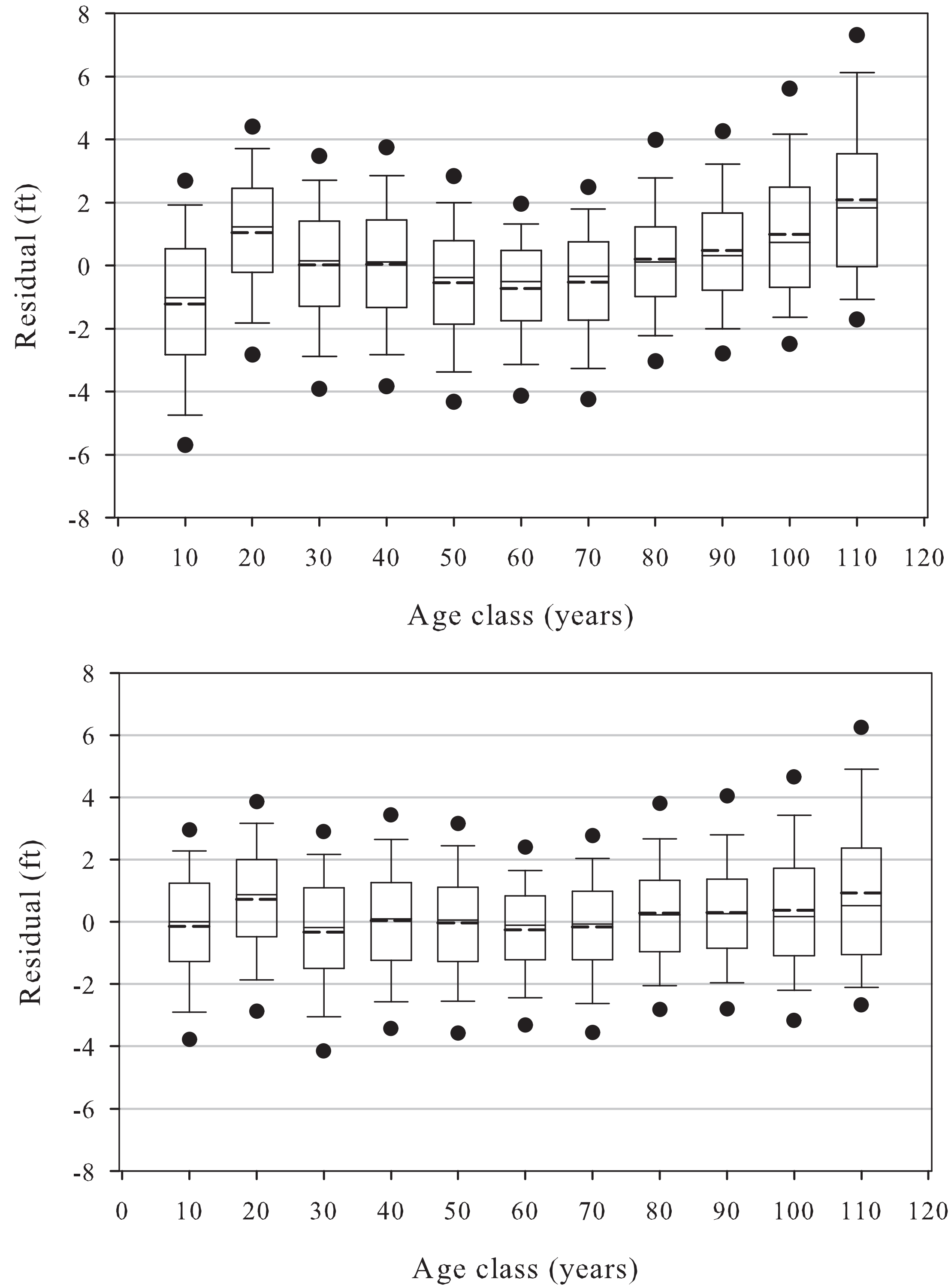

Figure 1. Box plot of residuals by breast height age class (10 = 5–14, 20 = 15–24, etc.) for Equation 2 (top) and Equation 4 (bottom). Boxes denote 25th and 75th percentiles, whiskers 10th and 90th percentiles, and dots 5th and 95th percentiles. Median and mean are represented by solid and dashed lines, respectively.

models that were polymorphic, could exhibit nearly linear rapid growth early, and did not flatten too much at older ages. Performance of models was improved by not considering height development between germination and 4.5 ft. The actual length of time between emergence and attainment of 4.5 ft in height was not measured on sample trees. Initial attempts to use a constant or fitted intercept for this period of growth indicated that it was both variable and correlated with site. Height growth above 4.5 ft and breast height age were used in fitting models to remove the problem of describing early growth.

Site index models examined were dynamic equations derived using the generalized algebraic difference approach (GADA). The theory of this approach, desirable characteristics of such derivations, and statistical considerations are discussed in detail by Cieszewski

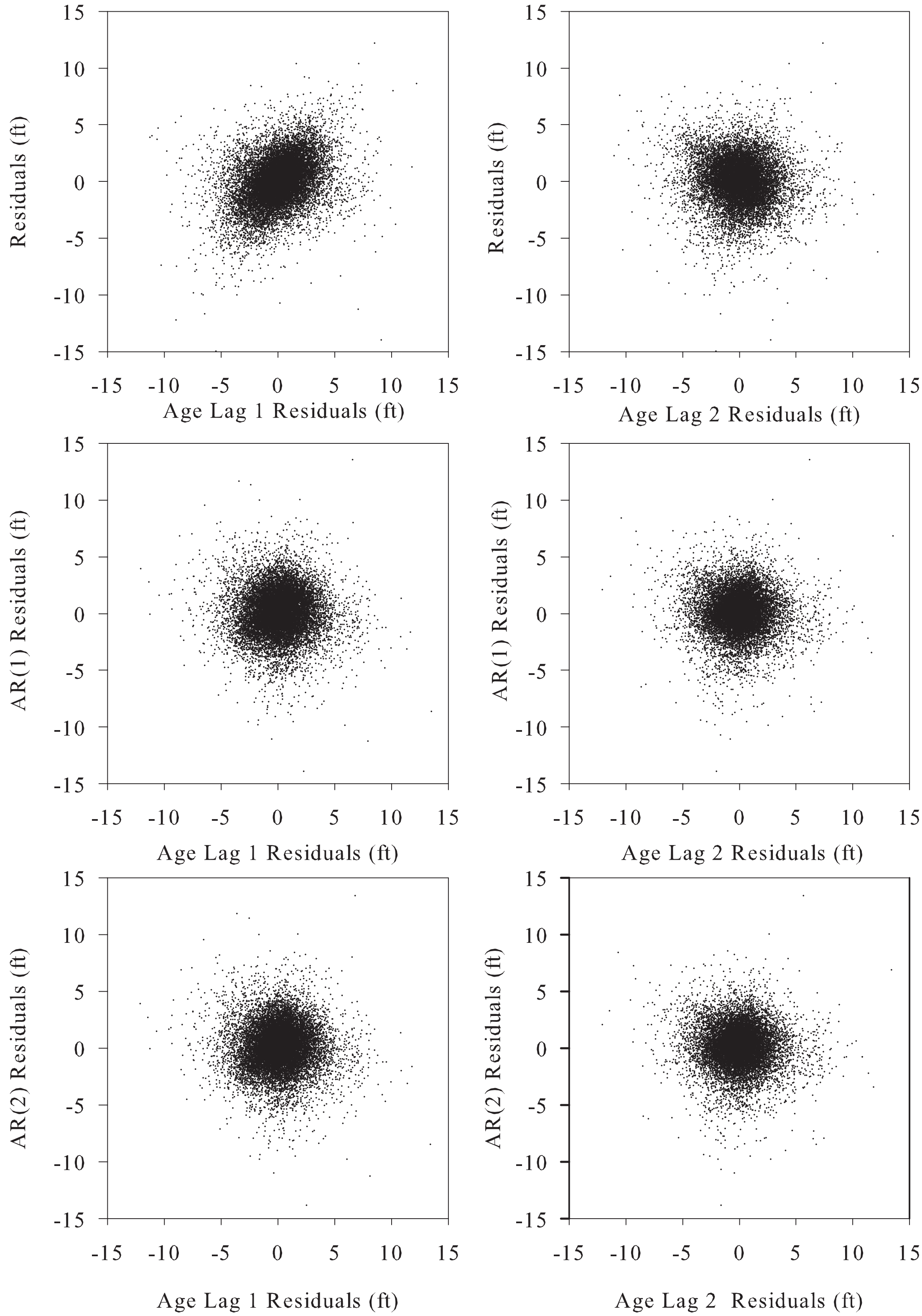

Figure 2. Height residuals versus 1 and 2 interval lagged age residuals for Equation 4. Residuals are plotted without autocorrelation parameters, with first-order (AR(1)) and second-order (AR(2)) autoregressive error structure.

and Bailey (2000). In summary, this method expands a base equation $Y = f(t)$ that describes growth as a function of time (age), t, into an explicit three-dimensional system $Y = f(t, X)$, where X is a theoretical growth intensity variable that describes how growth is related to site-specific dynamics. The change of site-specific parameters across site is defined as a function of X. Since X cannot be measured directly, it is parameterized in terms of initial conditions Y_0 and t_0. The resulting models are base-age invariant flexible functions with variable asymptotes that can be polymorphic. These models have been used for many species, including loblolly pine (Diéguez-Aranda et al. 2006), subalpine fir (Cieszewski 2003), and Douglas-fir (Cieszewski 2001).

Table 3. Comparison of parameter estimates for equation 4 with no autoregressive error, first-order autoregressive (AR1), and second-order autoregressive (AR2) error structures.

AR error structure	Parameter	Estimate	SE[a]	t value	Approximate pr. $>\|t\|$
None	b_1	73.327	1.538	47.7	0.001
	b_2	1877.54	79.31	23.7	0.001
	b_3	1.226	0.005	257.6	0.001
AR1	b_1	77.080	1.637	47.1	0.001
	b_2	1723.39	86.62	19.9	0.001
	b_3	1.235	0.005	228.9	0.001
	ρ_1	0.4744	0.0087	54.81	0.001
AR2	b_1	77.385	1.642	47.1	0.001
	b_2	1708.07	87.11	19.6	0.001
	b_3	1.236	0.005	226.3	0.001
	ρ_1	0.4881	0.0096	50.86	0.001
	ρ_2	0.0318	0.0114	2.77	0.006

[a] AR, autoregressive; SE, standard error.

Table 4. Root mean square error (RMSE), coefficient of multiple determination (R squared), log likelihood (LL), Akaike's information criteria (AIC), and Durbin-Watson test statistic for Equation 4 fit with different autoregressive error structures.

	Autoregressive error structure		
Statistic	None	AR1	AR2
RMSE	2.4270	2.2678	2.2676
R-squared	0.9872	0.9888	0.9888
LL	−43,232	−41,907	−41,904
AIC	93,003	90,356	90,350
Durbin-Watson[a]	0.95	1.46	1.47

[a] Durbin-Watson statistic calculated using panel data formula.

Many models were rejected because of their inability to describe rapid early height growth and the asymptotic pattern in older stands. Polynomial models were not used because a major objective of this project was to improve the performance of models for both young and old stands through the use of models with reasonable biological patterns of growth. Two final models were selected for comparison. The first is based on the function proposed by von Bertalanffy (1949, 1957) and used by Richards (1959) that has been widely used in forestry,

$$Y = a_1(1 - \exp(-a_2 t))^{a_3}, \tag{1}$$

where a_1 is an asymptote, a_2 is a rate parameter, and a_3 is a pattern parameter. Examination of various algebraic difference approach formulations, in which only one parameter is related to site, indicated that models that related the rate or asymptote parameter to site were not adequate and that more than one parameter needed to be a function of site characteristics. A GADA formulation for Equation 1 in which the asymptote and pattern parameters are related to site and related to each other by a linear function is as follows (Krumland and Eng 2005).

$$Y = Y_0\left(\frac{1 - \exp(-b_1 t)}{1 - \exp(-b_1 t_0)}\right)^{(b_2 + b_3/X_0)}, \tag{2}$$

$$X_0 = 0.5(\ln Y_0 - b_2 L_0 + \sqrt{(\ln Y_0 - b_2 L_0)^2 - 4 b_3 L_0}),$$

$$L_0 = \ln(1 - \exp(-b_1 t_0)).$$

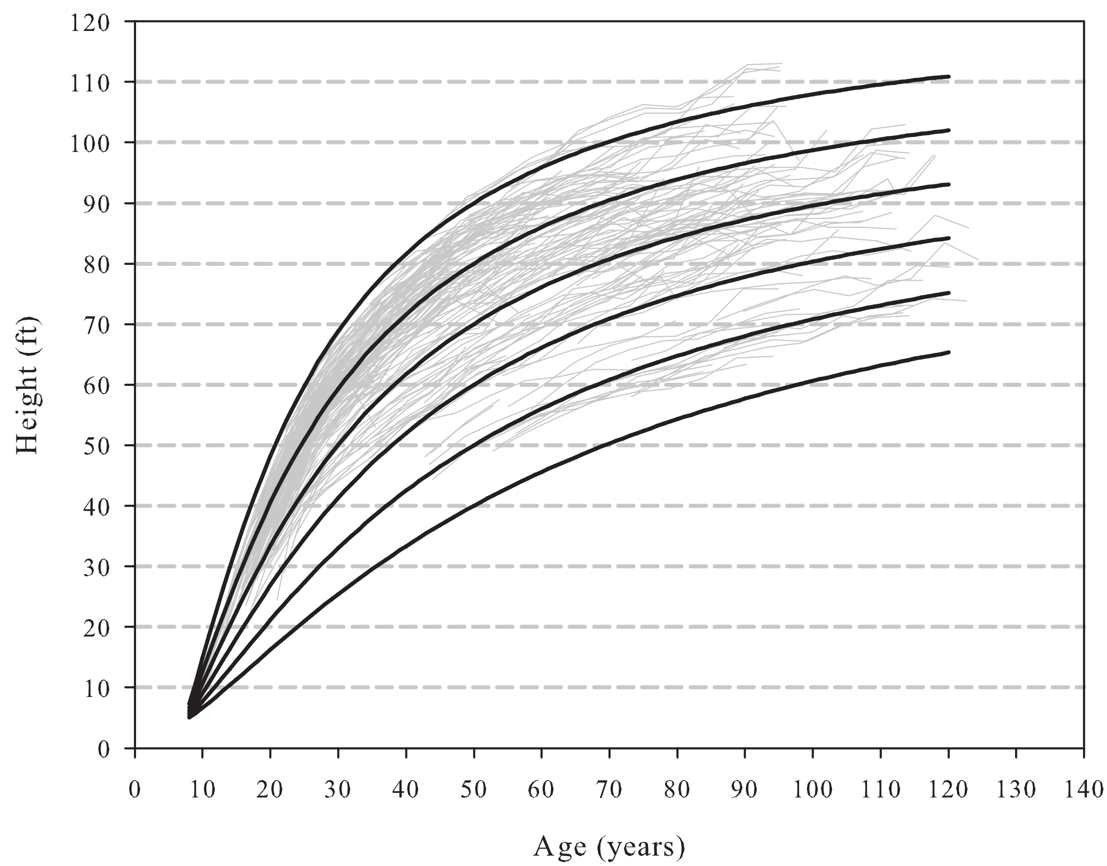

Figure 3. Fitted curves using Equation 9 with S_{43+7} (base age 50 with 7 years for trees to reach 4.5 ft) for site indices of 40, 50, 60, 70, 80, 90 ft overlaid with plot average height–age trajectories.

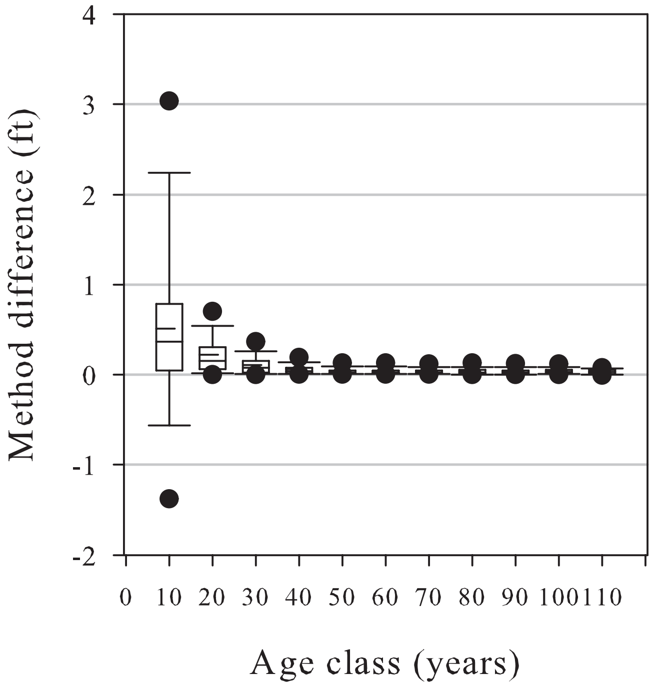

Figure 4. Difference in estimated site index methods by age class. Difference is averaged tree site index estimates on a plot minus site index estimated using average plot age and height.

The second model is based on the Hossfeld (1822) model:

$$Y = \frac{a_1}{1 + a_2 t^{-a_3}}, \tag{3}$$

which has been used with the GADA formulation by Cieszewski (2002, 2003) and adopting the formulation in Diéguez-Aranda et al. (2006) as follows.

$$Y = \frac{b_1 + X_0}{1 + (b_2/X_0)t^{-b_3}}, \tag{4}$$

$$X_0 = 0.5(Y_0 - b_1 + \sqrt{(Y_0 - b_1)^2 + 4b_2 Y_0 t_0^{-b_3}}).$$

Analysis

Estimation methods for permanent plot measurements usually use the average height of dominant and codominant trees on each plot as the experimental unit in the curve fitting process. The method here used individual-tree data as the experimental unit because ring-count at dbh varies within plots for natural longleaf pine.

The statistical model is as follows:

$$Y = f(Y_0, b_1, b_2, b_3, t_0, t) + e_{ij}, \tag{5}$$

where $Y_0 = Y_{0i}p_i$ is a tree specific parameter estimated for each tree, with $p_i = 1$ for tree i, 0 otherwise; and e_{ij} is the error for measurement j on tree i that is assumed to be identically and independently distributed with mean 0. The estimation of Y_{0i} as a unique parameter for each series of measurements on an individual using dummy variables (p_i) is a technique described by Cieszewski et al. (2000) in which the model is not restricted to pass through an observed specific point at base age. The use of a base-age invariant model and base-age invariant method for estimating parameters are used here in the sense of Bailey and Clutter (1974). For these data, Y is total tree height − 4.5 ft, and Y_0 is the estimated total height − 4.5 ft at a base age ring-count of 50 years (t_0). This method simultaneously estimates fixed effects (b_1, b_2, b_3) that are the same for all trees and random effects (Y_{0i}) that are unique for each tree.

Since serial correlation is expected for repeated measurements on the same individual, autocorrelation was modeled as a stationary autoregressive process, AR(n), as described by Cieszewski (2003). First- and second-order autoregressive error terms were tested as follows:

$$\text{AR(1):} \quad e_{ij} = d_1\rho_1 e_{ij-1} + \varepsilon_{ij} \tag{6}$$

$$\text{AR(2):} \quad e_{ij} = d_1\rho_1 e_{ij-1} + d_2\rho_2 e_{ij-2} + \varepsilon_{ij} \tag{7}$$

where e_{ij} is the jth ordinary residual for a measurement on the ith individual, $d_1 = 1$ for $j > 1$ and 0 for $j = 1$, $d_2 = 1$ for $j > 2$ and 0 for $j \leq 2$, and ρ_1, ρ_2 are autoregressive parameters to be estimated. Models were fit using the SAS/ETS model procedure (SAS Institute, Inc., 2004).

The use of individual-tree data instead of plot averages is one of utility for this data set. The usual approach for developing site index equations is to use plot average height growth series with approximately the same number of sample trees per plot such that the error term in Equation 5 has the desired properties. The utility of a site index function is based on the theoretical construct that the global model parameters are the same for all plots. It is difficult to construct a situation where this does not also imply that the global parameters are the same for individual trees. The methodology used here was considered the best way to use data from this rather large data set, both in number of plots and number of trees, to construct a site index equation.

The use of plot averages is problematic for this data set. The number of sample trees varies by plot and varies by age for a given plot. The plot average of t (age) is not an average of trees of the same age for a given measurement period and its distribution depends on the variation of age within a plot. The plot average of Y (height) is based on a sample of trees that differ in age and a sample that is changing through time because of thinning from below. One method for handling these issues would be to select a small subset of trees on each plot that were present at all evaluations and were of the same age. An attempt to select such a sample resulted in a rather small sample of trees that were not truly representative of actual stand conditions.

The individual-tree approach used here resolves the problems of measurement age and removal of trees in this long-term data set but does not address correlation among trees on a plot. A multilevel, nonlinear model approach that accounts for nested sources of variability might be a more reasonable statistical model, but it is not clear how to use such a model with this data set. In the current approach, the estimation of a fixed effect parameter for each individual tree given a set of global parameters for all trees provides an independent estimate of site index for each tree. However, the additional information of correlation among trees on a plot is not used. One effect of within plot correlation on the estimate of global parameters is that of weighting the influence of each plot with respect to sample size, in this case, that of weighting estimates toward younger stands. This methodology avoids parameter bias due to age structure and thinning, defines reasonable patterns of growth by choice of a mathematical model, and uses a large sample of trees and

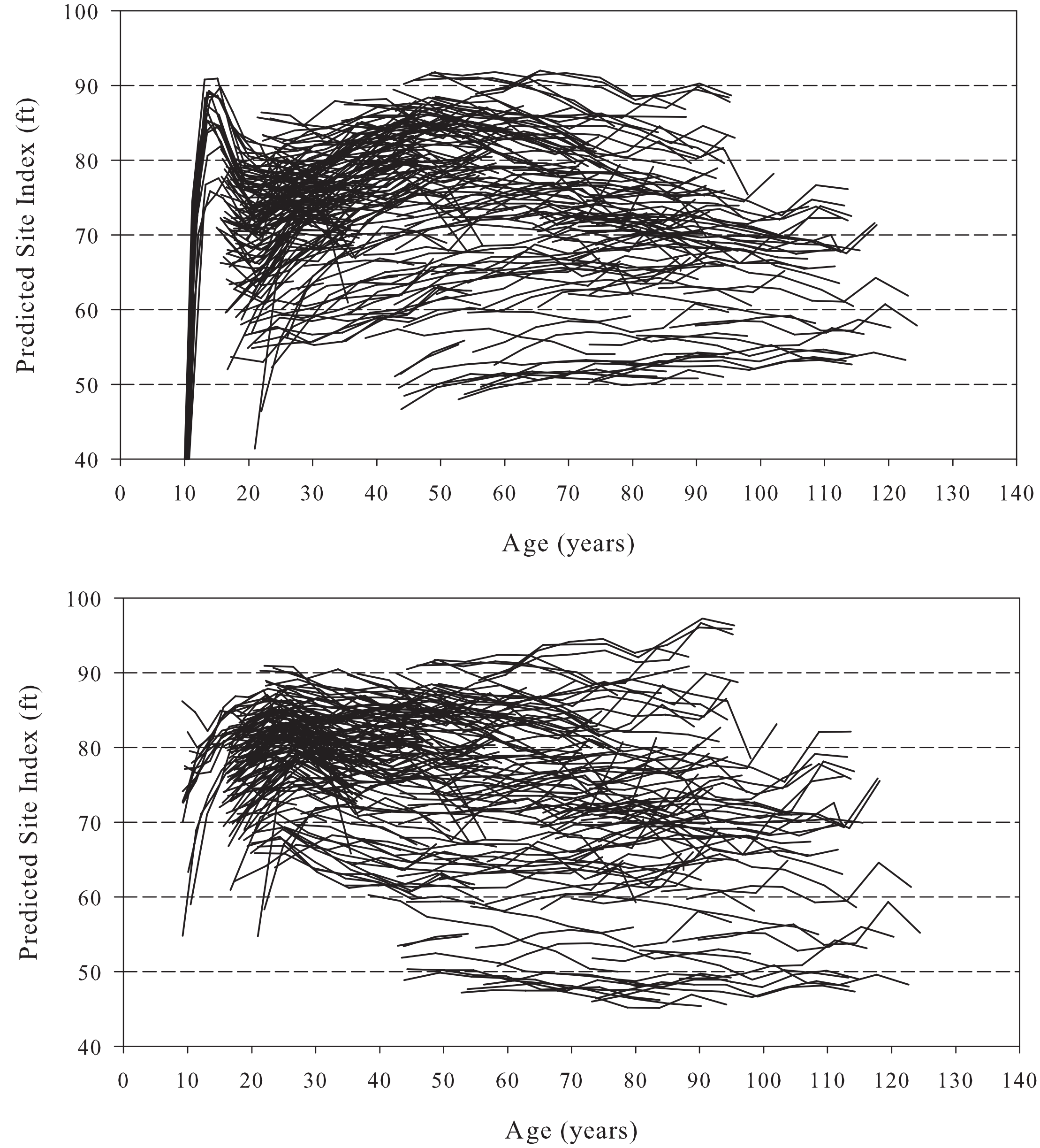

Figure 5. Plot-level estimated site index trajectories using Farrar's 1981 equation (top) and Equation 9 (bottom).

plots to characterize the population. The result is a site index equation that is useful and describes observed patterns of growth.

Model Comparison

Initial comparison of Equations 2 and 4 was performed to examine model performance over the range of ages. Scatter plots were not useful with more than 19,000 observations. Instead, box plots of residuals by age class were compared, and standard deviations of residuals were calculated by age class using models without autocorrelation terms included. Equation 4 performed better than Equation 2 in the sense that average residuals where closer to 0 throughout the age class range (Figure 1). This was particularly true for the young and old age classes. Equation 2 had difficulty in prediction for trees less than 25 years (ring count) and did not do well in predicting continued but slow growth for trees older than 75 years. Equation 4 performed reasonably well throughout the range, with underprediction for age classes 20 and 110. It was observed that a small percentage of trees in the 20-year age class surge in height. At ages over 80 years, crowns have flattened, and height patterns are the combination of slow growth, averaging 1–2 ft over a 5–10-year period, and leader breakage. The model average includes negative growth on some trees due to leader breakage for older trees.

Equation 4 was selected for further analysis because of its better performance. The interquartile ranges of residuals for this equation in Figure 1 were comparable among age classes. Residual standard deviations ranged from 1.8 to 2.2 ft for age classes 30–90 years, 2.3 and 2.4 ft for age classes 10 and 20, and 2.3 and 2.8 ft for age classes 100 and 110. Further analysis of Equation 4 involved the comparison of this model with respect to serial correlation.

Serial correlation was examined using scatter plots of age lagged residuals, examination of fit statistics for models with different error structures, and the panel data Durbin-Watson test statistic. Plots of age lag 1 and age lag 2 residuals for Equation 4 (Figure 2) without autoregressive parameters detected a pattern of positive correlation with age lag 1 residuals. This was not apparent for age lag 1 and age lag 2 residuals with AR1 or AR2 error structures. Global parameters

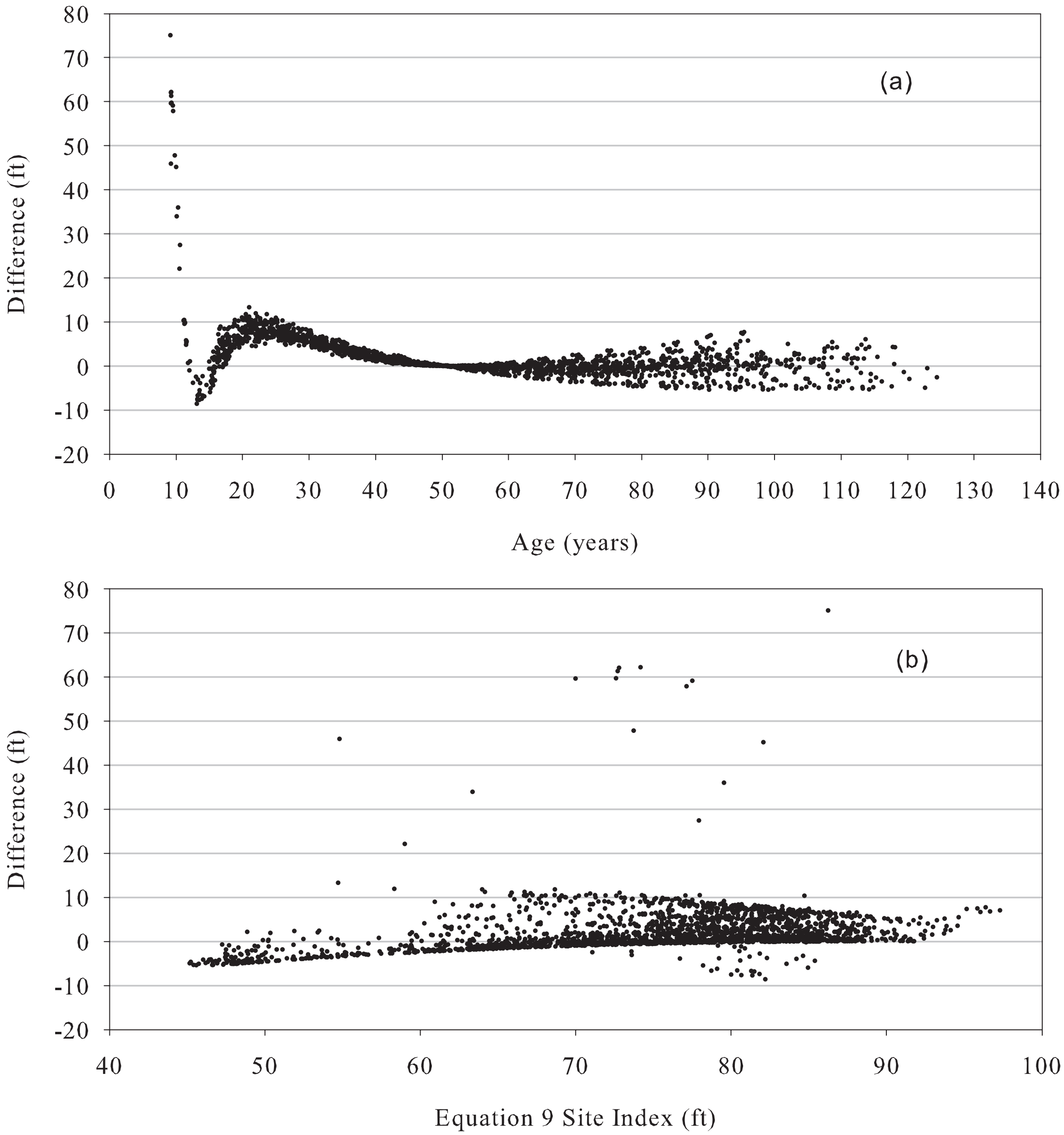

Figure 6. Differences in estimated site index on permanent measurement plots using Farrar's 1981 model and Equation 9 compared by age (a) and Equation 9 site index (b). Differences were computed as Equation 9 site index minus Farrar 1981 site index.

(Table 3) are very similar for the AR1 and AR2 models. The assumptions for the panel data Durbin-Watson statistic (error is normally distributed) are likely violated with this data but values closer to 2 should be better. This statistic improved from 0.95 to 1.46 and 1.47 for the AR1 and AR2 models, respectively. Fit statistics (Table 4) are essentially the same for both the AR1 and AR2 models. The log likelihood (larger is better) and Akaike's information criteria (smaller is better) (Akaike 1974) indicated a small improvement using the AR2 model. This improvement was not enough to adopt the more complex AR2 error structure over AR1. The AR1 model was adopted for further examination.

Equation 4 models height growth from the time a tree reaches 4.5 ft. The base age–invariant Equation 4 can be modified to account for age when a tree reaches 4.5 ft for height predictions, as

$$H = 4.5 + \frac{b_1 + X_0}{1 + (b_2/X_0)(\text{age} - G)^{-b_3}} \tag{8}$$

$$X_0 = 0.5((S_{rc+G} - 4.5) - b_1 + \sqrt{((S_{rc+G} - 4.5) - b_1)^2 + 4b_2(S_{rc+G} - 4.5)(B_{rc+G} - G)^{-b_3}}),$$

and site index prediction as

$$S_{rc+G} = 4.5 + \frac{b_1 + X_0}{1 + (b_2/X_0)(B_{rc+G} - G)^{b_3}} \tag{9}$$

$$X_0 = 0.5(H - 4.5 - b_1 + \sqrt{(H - 4.5 - b_1)^2 + 4b_2(H - 4.5)(\text{age} - G)^{-b_3}}),$$

where age is stand age in years, G is the age at which trees reach 4.5 ft, S is site index, B is site index base age, H is total height, and global parameters $b_1 = 77.080$, $b_2 = 1723.39$, $b_3 = 1.235$. The tree-specific and autocorrelation parameters are not included. Subscripts for S and B indicate how base age is referenced, with S_{43+7} indicating base age 50 years with a 43-year dbh ring count and 7 years to reach

4.5 ft (G). In practice, G can be measured or assumed. S_{50+0} can be used as a measure of site potential to compare stands that vary with respect to G if age_{bh} is known. Equation 8 predicted values for site indices of 40–90 ft are plotted with average plot height-age trajectories in Figure 3 using the traditional 50-year base age with 7 years to reach 4.5 ft in height (S_{43+7}).

Discussion and Conclusion

Previous site index models used plot average age, plot average height, and an assumed length of grass stage. These variables, and the fitted equation, would necessarily be influenced in some unknown way by removal of sample trees by thinning, age variation within a given plot (especially in young stands), and variation in the grass stage period. Equations 8 and 9 were developed using tree-specific parameters such that single tree height-age series were used to estimate the global model parameters that are not dependent on early development.

The proper way to use Equation 9 is to average site index calculated for each tree as opposed to using average plot age and height. In practice, the use of plot average age and height results in similar estimates for older stands with some bias in young stands. Differences between average tree site index and site index based on plot average age and height were less than 1 ft for stands over 15 years of age with this data set (Figure 4).

Performance of Equation 9 was compared with the original site index equation (Farrar 1981) by examining prediction trajectories on sample plots. Site indices using Equation 9 were the average of individual tree estimates using base age 50_{43+7} as is assumed by Farrar's equation. Ideally, site index estimates at different ages should remain close for a given plot (horizontal trajectories). Plotted trajectories (Figure 5) clearly show the estimation of site index in young stands is improved by Equation 9 but that estimation of young stand site index is still subject to some error. Outside the young stand age range, it is difficult to conclude that Equation 9 has improved estimates with both models showing similar patterns of longitudinal development. Equation 9 trajectories appear more horizontal for both high and low site indices at older ages. Equation 9 estimates, based on longer-term data, also differ in absolute terms. Equations differ for young ages, are similar for ages close to 50, and depart from each other at older ages (Figure 6a). Differences plotted against Equation 9 site index predictions (ignoring the large differences that are likely for young age estimates) suggest that Equation 9 expands the range of site index. Lower site indices are predicted for many plots on poorer sites and higher site indices for many plots on higher sites (Figure 6b). A definitive method of evaluating Equation 9 performance is not possible because of the lack of a definitive site index value for each plot in thinned stands where trees vary with respect to ring count.

This long-term permanent plot data has allowed the development of a site index equation for stands up to 120 years of age. The use of a base-age invariant equation that allows the input of the number of years it takes trees to reach 4.5 ft may allow Equations 8 and 9 to be more generally applied if the concept of site index referenced by ring count age and years to reach 4.5 ft is adopted for longleaf pine. Since these equations are based on height growth once trees reach 4.5 ft, they may prove to be more universal if the pattern of height growth above 4.5 ft is similar for longleaf stands of different origin or geographical locations. This hypothesis remains to be tested.

The discrete grass stage of longleaf complicates the selection of a site index reference age. The site index equation presented here would not be necessary for other species because a defined base age would be chosen. In this case, a base age reference is dependent on the length of the grass stage. The nontraditional use of ring count age at dbh for both longleaf pine site index and inputs into growth and yield models would improve the clarity of longleaf pine site quality estimates and growth and yield equations that use stand height or site index as model variables. In this case, the use of S_{25+0} or S_{50+0} would be preferred.

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